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Goldstone's Theorem on a Light-Like Plane

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Abstract I review various aspects of chiral symmetry and its spontaneous breaking on null planes, including the interesting manner in which Goldstone's theorem is realized and the constraints that chiral symmetry imposes on the null-plane Hamiltonians. Specializing to QCD with N massless flavors, I show that there is an interesting limit in which the chiral constraints on the null-plane Hamiltonians can be solved to give the spin-flavor algebra $SU(2N)$, recovering a result originally found by Weinberg using different methods.

Keywords Null-plane dynamics · Chiral symmetry · Goldstone's theorem

1 Introduction

Understanding of QCD as the correct underlying theory of the strong interaction relies on the interpretation provided by the parton model, which can be formulated in a frame-independent manner by quantizing QCD on a null plane or light front [1; 2]. However, until recently, a missing link in the null-plane description has been the lack of a model-independent description of spontaneous chiral symmetry breaking, which one would think is an essential non-perturbative ingredient in the matrix elements of local (and nonlocal) operators that appear in the partonic description. In particular, a formulation of Goldstone's theorem that does not depend on the formation of symmetry-breaking condensates has been lacking. And the manner in which fundamental QCD relations like the Gell-Mann-Oakes-Renner relation, which involve symmetry breaking condensates, are realized on the null-plane has caused considerable confusion. Recent work [3] has clarified these issues and several aspects of chiral symmetry breaking on null-planes will be reviewed here.

Chiral constraints on low-energy QCD observables are usually viewed in the context of chiral perturbation theory, which through fluctuations of the chiral condensate, allows one to calculate the long-distance Goldstone boson contributions to hadronic observables in a perturbative expansion in Goldstone boson masses and momenta. However, as pointed out by Weinberg long ago [4], in addition to these "dynamical" consequences of chiral symmetry in the infrared that are accessed using chiral perturbation theory, there are additional consequences of chiral symmetry that are "algebraic" in nature, and are generally expressed as sum rule constraints on observables that arise from specific assumptions about the asymptotic behavior of scattering amplitudes. The null-plane chiral constraints that are discussed here are precisely of the "algebraic" type found by Weinberg who used a completely different viewpoint that does not rely on null-plane quantization. This is gratifying since, at the end of the day, physics does not depend on the particular frame, coordinates or quantization surfaces that are chosen.

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2 Null-Plane Hamiltonians

In the usual instant-form quantization, the four dynamical generators are the energy, or time-evolution operator, and the boosts. However, as the boosts are not associated with any observable, in the instant-form, the focus is on obtaining the Hamiltonian energy operator. By contrast, on a null-plane there are three dynamical generators, corresponding to the light-cone time evolution operator, or energy, and the transverse components of the spin operator [5]. As the spin is an important observable, we refer to the three dynamical generators as Hamiltonians. Remarkably, on a light-like plane, all dynamical information of a Poincaré invariant theory of quantum mechanics is contained in the three reduced Hamiltonians $M\mathcal{J}_r$ (with $r = 1, 2$) and M^2 which encode the spin content and spectrum, respectively [6]. These reduced Hamiltonians commute with six of the kinematical generators of the Poincaré algebra and satisfy the $U(2)$ algebra together with the seventh kinematical generator, $\mathcal{J}_3$:

$$\begin{aligned} [\,\mathcal{J}_3\,,\,M\mathcal{J}_r\,] &= i\,\epsilon_{rs}M\mathcal{J}_s \quad , \quad [\,\mathcal{J}_3\,,\,M^2\,] = 0\;; \\ [\,M\mathcal{J}_r\,,\,M\mathcal{J}_s\,] &= i\,\epsilon_{rs}M^2\mathcal{J}_3 \quad , \quad [\,M^2\,,\,M\mathcal{J}_r\,] = 0\;. \end{aligned} \tag{1}$$

This decoupling of the kinematical and dynamical generators is an important property of null-plane quantization, particularly as regards the issue of chiral symmetry breaking, as we know that all chiral symmetry breaking is necessarily contained in the three reduced Hamiltonians.

As spin is dynamical on the null-plane, *a priori* one does not expect that the null-plane description in terms of Lagrangian field theory will exhibit manifest Lorentz invariance. Spin takes its proper form only when interactions are explicitly taken into account –that is, when the dynamics of the system are fully solved. Hence the arrangement of the spectrum of the theory into representations of the Lorentz group is evident only in the solution of the theory. In particular, while the spin of the system is usually given by the sum of the spins of the constituents, here that is no longer the case. The spin of the system is given by the sum of the spins of the constituents as well as by the interactions among the constituent spins.

3 Null-Plane Chiral Symmetry

Consider a system whose action has an exact $SU(N)_R \otimes SU(N)_L$ symmetry, with Noether currents given by $\tilde{J}^\mu_\alpha(x)$ and $\tilde{J}^\mu_{5\alpha}(x)$, and whose internal charges, defined by

$$\tilde{Q}_\alpha \;=\; \int dx^-\, d^2\boldsymbol{x}_\perp\, \tilde{J}^+_\alpha(x^-, \mathbf{x}_\perp)\;; \tag{2}$$

$$\tilde{Q}^5_\alpha \;=\; \int dx^-\, d^2\boldsymbol{x}_\perp\, \tilde{J}^+_{5\alpha}(x^-, \mathbf{x}_\perp)\;, \tag{3}$$

satisfy the chiral algebra:

$$\begin{aligned} [\,\tilde{Q}^\alpha\,,\,\tilde{Q}^\beta\,] = i\,f^{\alpha\beta\gamma}\,\tilde{Q}^\gamma \;\;,\;\; [\,\tilde{Q}^\alpha_5\,,\,\tilde{Q}^\beta\,] &= i\,f^{\alpha\beta\gamma}\,\tilde{Q}^\gamma_5\;; \\ [\,\tilde{Q}^\alpha_5\,,\,\tilde{Q}^\beta_5\,] &= i\,f^{\alpha\beta\gamma}\,\tilde{Q}^\gamma\;. \end{aligned} \tag{4}$$

Here we assert that on a null-plane both types of chiral charges annihilate the vacuum. That is,

$$\tilde{Q}^\alpha\,|\,0\,\rangle \;=\; \tilde{Q}^\alpha_5\,|\,0\,\rangle \;=\; 0\;. \tag{5}$$

It is straightforward to verify these relations explicitly in null-plane QCD [3]. In general, this structureless nature of the null-plane vacuum arises because the null-plane momentum operator has a spectrum confined to the open positive half-line. (Here one should keep in mind that the correct treatment of the singularities that arise in null-plane quantization is a subtle mathematical issue [7].) The important consequence of this is that the null-plane vacuum is invariant with respect to the full $SU(N)_R \otimes SU(N)_L$ symmetry, even in the phase in which the symmetry is said to be spontaneously broken. In particular, this implies that there can be no vacuum condensates that break $SU(N)_R \otimes SU(N)_L$ on a null-plane [3]. If the chiral symmetry is not spontaneously broken then one expects that the chiral currents are conserved and the chiral charges commute with the reduced Hamiltonians. However, if the symmetry is spontaneously broken, then one must have

$$[\,\tilde{Q}^\alpha_5\,,\,M^2\,] \;\neq\; 0 \quad ; \quad [\,\tilde{Q}^\alpha_5\,,\,M\mathcal{J}_\pm\,] \;\neq\; 0\;, \tag{6}$$

where $\mathcal{J}_\pm \equiv \mathcal{J}_1 \pm i\mathcal{J}_2$. That is, given the invariance of the vacuum, symmetry breaking must be present in the Hamiltonians. The particular pattern of breaking depends on the assumed form of chiral symmetry breaking. In QCD with N flavors of massless quarks the Lie brackets among the reduced Hamiltonians and the chiral charges are easily evaluated to give [3]

$$\mathcal{P}^{\alpha\beta;\mu\nu}\,[\tilde{Q}_5^\mu\,,\,[\tilde{Q}_5^\nu\,,\,M^2]] \;=\; \mathcal{P}^{\alpha\beta;\mu\nu}\,[\tilde{Q}_5^\mu\,,\,[\tilde{Q}_5^\nu\,,\,M\mathcal{J}_\pm]] \;=\; 0\;, \tag{7}$$

where

$$\mathcal{P}^{\alpha\beta;\mu\nu} \;\equiv\; \delta^{\alpha\nu}\delta^{\beta\mu} \;-\; \frac{1}{N^2-1}\,\delta^{\alpha\beta}\delta^{\mu\nu} \;-\; \frac{N}{N^2-4}\,d^{\alpha\beta\gamma}d^{\mu\nu\gamma}\;. \tag{8}$$

These constraints imply that M^2 and $M\mathcal{J}_\pm$ transform as linear combinations of $(1,1)$, $(\bar{\mathbf{N}},\mathbf{N})$ and $(\mathbf{N},\bar{\mathbf{N}})$ representation of $SU(N)_R\otimes SU(N)_L$.

4 Goldstone's Theorem

The standard field-theoretic paradigm tells us that a classical symmetry of an action has three possible fates after quantization: the symmetry remains unbroken and the currents are conserved, the symmetry is spontaneously broken and once again the currents are conserved, or the symmetry is anomalous and the associated current is not conserved. The null plane realizes a fourth possibility: the symmetry is spontaneously broken and the associated current is not conserved. This pattern is a necessary consequence of the vacuum being invariant with respect to all internal symmetries on a null plane. Now if we add an explicit chiral symmetry breaking operator to the action, then, in general, one has

$$\partial_\mu \tilde{J}^\mu_{5\alpha}(x^-,\mathbf{x}_\perp,x^+) \;=\; \epsilon_\chi\,\tilde{P}_\alpha(x^-,\mathbf{x}_\perp,x^+)\;, \tag{9}$$

where ϵ_χ is a parameter that measures the amount of explicit chiral symmetry breaking that is present in the Lagrangian. Using the short hand,

$$|\,h\,\rangle \equiv |\,p^+\,,\,\mathbf{p}_\perp\,;\,\lambda\,,\,h\,\rangle\;, \tag{10}$$

for the momentum eigenstates, we take the matrix element of eq. 6 (left side) between momentum eigenstates, which gives [3]

$$\langle\,h'\,|[\,\tilde{Q}^5_\alpha(x^+)\,,\,M^2]|\,h\,\rangle \;=\; -2i\,p^+\,\epsilon_\chi\int dx^-\,d^2\boldsymbol{x}_\perp\,\langle\,h'\,|\tilde{P}_\alpha(x^-,\mathbf{x}_\perp,x^+)|\,h\,\rangle\;. \tag{11}$$

If the right hand side of this equation vanishes for all h and h', then there can be no chiral symmetry breaking of any kind, since —as we have argued— there can be no symmetry breaking condensates and therefore all symmetry breaking must reside in the Hamiltonians. Hence, in order that the chiral symmetry be spontaneously broken, the chiral current cannot be conserved and we have the following constraint [8; 9] in the limit where the explicit symmetry breaking is turned off, $\epsilon_\chi\to 0$:

$$\int dx^-\,d^2\boldsymbol{x}_\perp\,\langle\,h'\,|\tilde{P}_\alpha(x^-,\mathbf{x}_\perp,x^+)|\,h\,\rangle \longrightarrow \frac{1}{\epsilon_\chi}\;+\;\ldots\;, \tag{12}$$

where the dots represent other terms that are regular in the limit $\epsilon_\chi\to 0$. Now we will show that this condition implies the existence of N^2-1 Goldstone bosons. As in the instant-form formulation of chiral symmetry breaking, we can treat the operator $\tilde{P}_\alpha$ as an interpolating operator for Lorentz-scalar fields ϕ^i_α that carry the same quantum numbers, and we can write

$$\tilde{P}_\alpha(x) \;=\; \sum_i \mathcal{Z}_i\,\phi^i_\alpha(x) \tag{13}$$

where the $\mathcal{Z}_i$ are overlap factors. Now we can use standard technology, i.e. the reduction formula, to relate the matrix elements of this operator between physical states to transition amplitudes. The S-matrix element for the transition $h(p)\to h'(p')+\phi^i_\alpha(q)$ is defined as

$$\begin{aligned}\langle\,h'\,;\,\phi^i_\alpha(q)\,|S|\,h\,\rangle &\equiv i(2\pi)^4\,\delta^4(\,p\,-\,p'-q)\,\mathcal{M}^i_\alpha(\,p',\,\lambda',\,h'\,;\,p,\,\lambda,\,h\,)\;;\\ &= i\int d^4x\,e^{-iq\cdot x}\,\left(-q^2+M^2_{\phi^i}\right)\,\langle\,h'\,|\,\phi^i_\alpha(x)\,|\,h\,\rangle\end{aligned} \tag{14}$$

where $\mathcal{M}^i_\alpha$ is the Feynman amplitude and in the second line the reduction formula has been used. It follows that

$$\langle\, h' \,|\, \phi^i_\alpha(x) \,|\, h \,\rangle = -e^{iq\cdot x}\, \frac{1}{q^2 - M^2_{\phi^i}}\, \mathcal{M}^i_\alpha(q)\ . \tag{15}$$

Using this result, together with eq. 13, in eq. 11 gives

$$\begin{aligned}\langle\, h' \,|[\, \tilde{Q}^5_\alpha(x^+)\,,\, M^2]|\, h\,\rangle &= 2i\, p^+\, (2\pi)^3\, \delta(\, q^+\,)\, \delta^2(\, \mathbf{q}_\perp\,)\, e^{ix^+q^-} \sum_i \frac{\epsilon_\chi\, \mathcal{Z}_i}{2q^+q^- \,-\, \mathbf{q}^2_\perp - M^2_{\phi^i}}\, \mathcal{M}^i_\alpha(q)\ ;\\ &= \,-\, 2i\, p^+\, (2\pi)^3\, \delta(\, q^+\,)\, \delta^2(\, \mathbf{q}_\perp\,)\, e^{ix^+q^-} \sum_i \frac{\epsilon_\chi\, \mathcal{Z}_i}{M^2_{\phi^i}}\, \mathcal{M}^i_\alpha(q^-)\ .\end{aligned} \tag{16}$$

In order that the right hand side of this equation be non-zero in the symmetry limit, there must be at least one field ϕ^i_α whose mass-squared vanishes proportionally to the symmetry-breaking parameter ϵ_χ as $\epsilon_\chi \to 0$. We will denote this field as $\pi^\alpha \equiv \phi^1_\alpha$ with

$$M^2_\pi \;=\; c_p\, \epsilon_\chi\ , \tag{17}$$

where c_p is a constant. There are therefore $N^2 - 1$ massless fields π_α in the symmetry limit, which are identified as the Goldstone bosons. It is important to note that this proof of Goldstone's theorem relies entirely on physical matrix elements. That is, unlike the usual textbook proof of Goldstone's theorem, there is no need here to assume the existence of vacuum condensates that transform non-trivially with respect to the chiral symmetry group. Of course, in instant-form quantized QCD, we know that the proportionality constant c_p in eq. 17 contains the quark condensate. We will return to this point in the next section.

Writing $\tilde{P}_\alpha \;=\; \mathcal{Z}\, \pi_\alpha \;+\; \ldots$ where the dots represent other (non-Goldstone) boson fields, and

$$\langle\, h' \,|\, \partial_\mu\, \tilde{J}^\mu_{5\alpha}(x) \,|\, h\,\rangle = \langle\, h' \,|\, \bar{\mathcal{Z}}\, M^2_\pi\, \pi_\alpha(x) \,|\, h\,\rangle\ , \tag{18}$$

where $\bar{\mathcal{Z}} \equiv \mathcal{Z}/c_p$. It is now a standard exercise to determine the overlap factor. Defining the Goldstone-boson decay constant, F_π, via

$$\langle\, 0 \,|\, \tilde{J}^\mu_{5\alpha}(x) \,|\, \pi_\beta\,\rangle \equiv -i\, p^\mu\, F_\pi\, \delta_{\alpha\beta}\, e^{ip\cdot x}\ , \tag{19}$$

where $|\, \pi_\beta\,\rangle \equiv |\, p^+\,,\, \mathbf{p}_\perp\,;\, 0,\, \pi_\beta\,\rangle$, one finds $\bar{\mathcal{Z}} = F_\pi$.

5 Null-Plane Condensates

In QCD with N degenerate flavors of quarks eq. 17 takes the form of the Gell-Mann-Oakes-Renner formula [10]

$$-\, M\, \langle\, \Omega \,|\, \bar{\psi}\psi \,|\, \Omega\,\rangle \;=\; \tfrac{1}{2}\, N\, M^2_\pi\, F^2_\pi \;\;+\; \ldots\ , \tag{20}$$

where M is the quark mass, ψ is the quark field, and $|\, \Omega\,\rangle$ is the instant-form vacuum. With respect to the instant-form chiral charges, Q^α_5, $\bar{\psi}\psi$ transforms as $(\bar{\mathbf{N}}, \mathbf{N}) \oplus (\mathbf{N}, \bar{\mathbf{N}})$ under $SU(N)_R \otimes SU(N)_L$. It is a simple textbook exercise to check this; one finds

$$[\, Q^\alpha_5\,,\, \psi\,] \;=\; -\gamma_5\, T^\alpha\, \psi\ , \tag{21}$$

where the T_α are $SU(N)$ generators. This transformation property implies that the quarks transform *irreducibly* with respect to $SU(N)_R \otimes SU(N)_L$; that is, $\psi_R \in (\mathbf{1}, \mathbf{N})$ and $\psi^\dagger_L \in (\bar{\mathbf{N}}, \mathbf{1})$. The claimed transformation property of $\bar{\psi}\psi$ then follows.

How then is this relation reconciled with our claim that there are no symmetry-breaking condensates on a null-plane? With respect to the null-plane (tilded) charges, one finds [11]

$$[\, \tilde{Q}^\alpha_5\,,\, \psi\,] \;=\; -\gamma_5\, T^\alpha\, \psi \;-\; i\, \gamma_5\, \gamma^+\, T^\alpha\, \frac{1}{\partial^+_M}\psi\ , \tag{22}$$

from which it follows that

$$\psi_R\,,\,\psi_L\;\in\;(\mathbf{1},\mathbf{N})\oplus(\mathbf{N},\mathbf{1})\quad,\quad\psi_R^\dagger\,,\,\psi_L^\dagger\;\in\;(\mathbf{1},\bar{\mathbf{N}})\oplus(\bar{\mathbf{N}},\mathbf{1})\;. \tag{23}$$

That is, the quarks transform *reducibly* with respect to $SU(N)_R\otimes SU(N)_L$. Because of this reducibility, the bilinear $\bar\psi\psi$ always contains the $SU(N)_R\otimes SU(N)_L$ singlet! Indeed, in terms of the dynamical quark field ψ_+ the null-plane expression of the Gell-Mann-Oakes-Renner relation can be formally written as [12]

$$M\,\langle\,0\,|\,i\,\bar\psi_+\gamma^+\frac{1}{\partial_M^+}\psi_+\,|\,0\,\rangle\;=\;\tfrac{1}{2}\,N\,M_\pi^2\,F_\pi^2\;\;+\;\ldots\;, \tag{24}$$

where now $|\,0\,\rangle$ is the null-plane vacuum state, and the nonlocal operator $1/\partial_M^+$ is defined in Ref. [3]. Hence we see that a chiral-symmetry breaking condensate in the instant-form formulation of QCD is replaced by a chiral-symmetry conserving condensate in the null-plane formulation. Of course both relations, eq. 20 and eq. 24, contain precisely the same physics, as they must.

6 Weinberg's Recovery of Spin-Flavor Symmetries

The goal of finding solutions of the algebraic system that mixes the chiral charges and the reduced Hamiltonians may seem hampered by the existence of no-go theorems that forbid the non-trivial mixing of space-time and internal symmetries. In the null-plane formulation, these no-go theorems are avoided because it is only the dynamical part, i.e. the Hamiltonians, of the null-plane Poincaré algebra that mix with the chiral symmetry generators [13].

While a general solution of the null-plane QCD operator algebra, given by eqs. 1, 4, and 7, is not known, there is a very-interesting limiting case in which the algebra yields an important and familiar solution. Here we will work with the QCD operator algebra. However, it is important to keep in mind that matrix elements of the operator relations between physical, hadronic states must be taken in order to extract observables. We first define

$$[\,\tilde{Q}_5^\alpha\,,\,M\,]\;\equiv\;\epsilon^\alpha\;, \tag{25}$$

and throw away terms of $\mathcal{O}(\epsilon)$. This implies that all chiral symmetry breaking must occur in the spin Hamiltonians, $M\mathcal{J}_\pm$. In this limit, the QCD operator algebra reduces to

$$[\,\mathcal{J}_i\,,\,\mathcal{J}_j\,]\;=\;i\,\epsilon_{ijk}\,\mathcal{J}_k \tag{26}$$

which generates $SU(2)$ spin, the $SU(N)_R\otimes SU(N)_L$ algebra of eq. 4, and the remaining non-trivial Lie bracket mixes the chiral generators and the spin Hamiltonians:

$$\mathcal{P}^{\alpha\beta;\mu\nu}\,[\tilde{Q}_5^\mu\,,\,[\tilde{Q}_5^\nu\,,\,\mathcal{J}_\pm]]\;=\;0\;. \tag{27}$$

Remarkably, this simplified algebra can be put into a familiar form. Consider an operator $G_{\alpha i}$, which transforms as an adjoint of $SU(N)$ and as a vector with respect to rotations. In general, the commutator of $G^{\alpha i}$ with itself may be expressed as

$$[\,G_{\alpha i}\,,\,G_{\beta j}\,]\;=\;i\,f_{\alpha\beta\gamma}\,\mathcal{A}_{ij,\gamma}\;+\;i\,\epsilon_{ijk}\,\mathcal{B}_{\alpha\beta,k}\;, \tag{28}$$

where $\mathcal{A}_{ij,\gamma}=\mathcal{A}_{ji,\gamma}$ and $\mathcal{B}_{\alpha\beta,k}=\mathcal{B}_{\beta\alpha,k}$. Now we identify $G^{\alpha 3}\;\equiv\;\tilde{Q}_5^\alpha$. Detailed consideration of the properties of $G^{\alpha\beta}$ determines $\mathcal{A}$ and $\mathcal{B}$ and finally leads to [14; 3]

$$[\,G_{\alpha i}\,,\,G_{\beta j}\,]\;=\;i\,\delta_{ij}\,f_{\alpha\beta\gamma}\,\tilde{Q}_\gamma\;+\;\frac{2}{N}\,i\,\delta_{\alpha\beta}\,\epsilon_{ijk}\,\mathcal{J}_k\;+\;i\epsilon_{ijk}\,d_{\alpha\beta\gamma}\,G_{\gamma k}\;, \tag{29}$$

which together with

$$[\,\tilde{Q}_\alpha\,,\,G_{\beta i}\,]\;=\;i\,f_{\alpha\beta\gamma}\,G_{\gamma i}\quad,\quad[\,\mathcal{J}_i\,,\,G_{\alpha j}\,]\;=\;i\,\epsilon_{ijk}\,G_{\alpha k}\;; \tag{30}$$

$$[\,\tilde{Q}_\alpha\,,\,\tilde{Q}_\beta\,]\;=\;i\,f_{\alpha\beta\gamma}\,\tilde{Q}_\gamma\quad,\quad[\,\mathcal{J}_i\,,\,\mathcal{J}_j\,]\;=\;i\,\epsilon_{ijk}\,\mathcal{J}_k \tag{31}$$

close to the algebra of the group $SU(2N)$. It is important to stress that this $SU(2N)$ symmetry is truly a dynamical symmetry; it is unrelated to the invariance of QCD in the non-interacting limit.

Importantly, this symmetry can be viewed as emerging in a particular limit of QCD. As the hadronic matrix element of eq. 25, $\langle h' | \epsilon^\alpha | h \rangle \sim M_h - M_{h'}$, and baryons within large-N_c multiplets have mass splittings that scale as $1/N_c$ [15], standard large-N_c QCD scaling rules suggest that for baryons $\epsilon^\alpha \sim 1/N_c$. Moreover, as the matrix element of chiral charges between baryon states scales as N_c, the $SU(2N)$ symmetry formally reduces to the contracted $SU(2N)$ symmetry [14] for baryons in the large-N_c limit, as must occur on general grounds [16; 17; 18].

It is instructive to consider a simple example in order to see how null-plane chiral symmetry constrains hadronic structure. Consider the case $N = 3$. Using the null-plane chiral transformation properties of the quarks [3]

$$\psi_{+R} \;=\; \psi_{+\uparrow} \;\in\; (\mathbf{1}, \mathbf{N}) \qquad , \qquad \psi^\dagger_{+R} \;=\; \psi^\dagger_{+\downarrow} \;\in\; (\mathbf{1}, \bar{\mathbf{N}}) \; ; \tag{32}$$

$$\psi_{+L} \;=\; \psi_{+\downarrow} \;\in\; (\mathbf{N}, \mathbf{1}) \qquad , \qquad \psi^\dagger_{+L} \;=\; \psi^\dagger_{+\uparrow} \;\in\; (\bar{\mathbf{N}}, \mathbf{1}) \; , \tag{33}$$

one sees that a $\lambda = 3/2$ baryon-like operator $\psi_{+\uparrow}\psi_{+\uparrow}\psi_{+\uparrow}$ transforms as $(\mathbf{1}, \mathbf{1})$, $(\mathbf{1}, \mathbf{8})$, or $(\mathbf{1}, \mathbf{10})$ with respect to $SU(3)_R \otimes SU(3)_L$. Therefore, if the baryon is a decuplet of $SU(3)_F$ with its $\lambda = 3/2$ part in the $(\mathbf{1}, \mathbf{10})$, then one readily checks that its $\lambda = 1/2$ component must transform as $(\mathbf{3}, \mathbf{6})$ or $(\mathbf{6}, \mathbf{3})$. However, the various helicity states are unrelated by chiral symmetry in itself. Strangely, it is the mixed Lie-bracket, eq. 27, the contribution of spontaneously broken chiral symmetry to the spin Hamiltonian, that relates the helicities! It is counter intuitive to have states that are unrelated in the symmetry limit, become part of a symmetry multiplet in the broken-symmetry limit. Nevertheless, this is precisely what occurs on the null-plane. Taking the $\lambda = 1/2$ baryon decuplet to transform as $(\mathbf{3}, \mathbf{6})$ together with an baryon octet spin-1/2, and with their negative-helicity partners in $(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{6}, \mathbf{3})$, together form the **56**-dimensional representation of $SU(6)$, which is the familiar ground-state baryon assignment in the non-relativistic quark model. The difference between what has been found here and the quark model is that the symmetry that arises from the null-plane algebra follows directly from QCD symmetries and their pattern of breaking. In particular, this symmetry has nothing to do with the non-relativistic limit.

In summary, beginning from the null-plane QCD operator algebra, the assumption that the chiral-symmetry breaking part of the null-plane reduced Hamiltonian, M^2, is small, implies all of the usual consequences of the non-relativistic quark model, but without the need to assume constituent quark degrees of freedom [14]. The next step is to investigate how the $\mathcal{O}(\epsilon)$ terms that were neglected modify the simple quark-model-like picture. For instance, it is clear that the nucleon null-plane wavefunction will then be a mixture of various representations which will *inter alia* constrain the spin content and spectrum of the light baryons.

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